Exercise 7

Find the area of a triangle bounded by the y-axis, the line $f(x) = 9 - \frac{6}{7}x$, and the line perpendicular to $f(x)$ that passes through the origin.

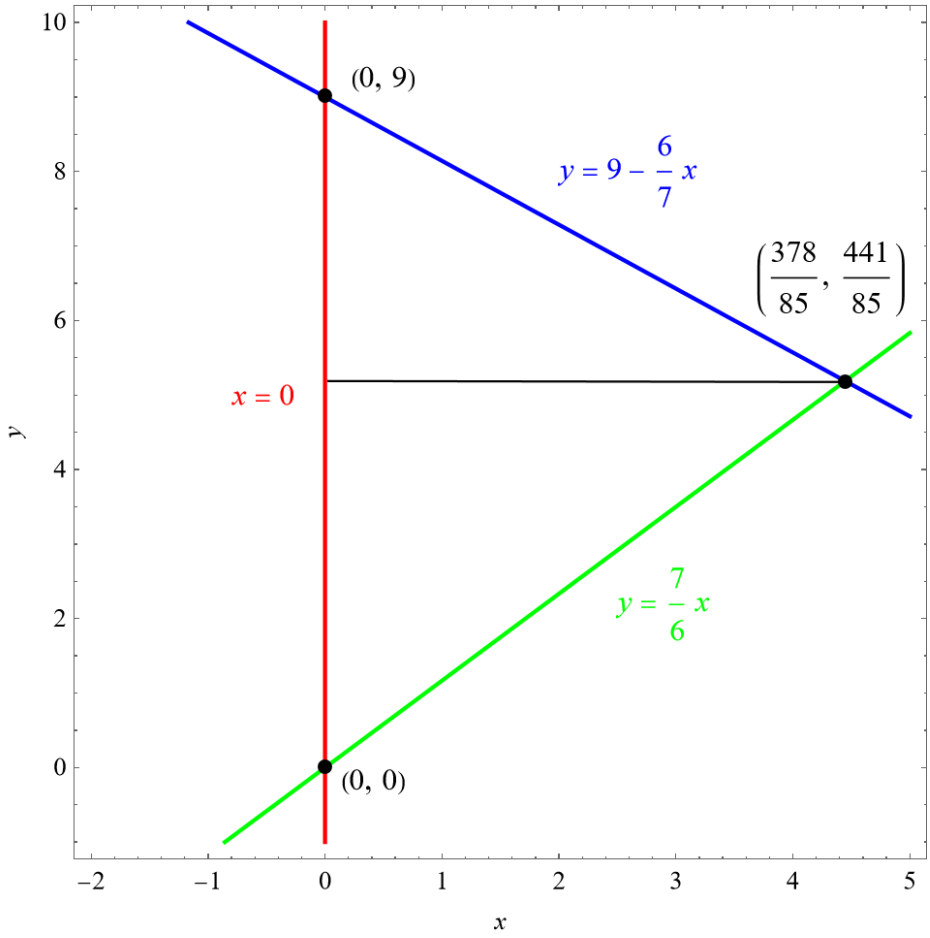
Solution

Start by writing equations of the lines that are given. The equation for the y-axis is $x = 0$, $y = 9 - \frac{6}{7}x$ is given, and the line perpendicular to $f(x)$ has the negative reciprocal slope $(7/6)$ with an equation given by the point-slope formula.

$$y - 0 = \frac{7}{6}(x - 0)$$

$$y = \frac{7}{6}x$$

Now graph all of them.



The area of the triangle is half the product of the base and height.

$$A = \frac{1}{2}bh = \frac{1}{2}(9)\left(\frac{378}{85}\right) = \frac{1701}{85}.$$

The point of intersection on the right is found by setting the two functions of x on the right equal to each other and solving for x.

$$\frac{7}{6}x = 9 - \frac{6}{7}x$$

$$\frac{7}{6}x + \frac{6}{7}x = 9$$

$$\frac{85}{42}x = 9$$

$$x = \frac{378}{85}$$

Plug this value of x into either of the two functions to determine the corresponding y-value.

$$y = \frac{7}{6}\left(\frac{378}{85}\right) = \frac{441}{85}$$

This means the intersection point on the right is $\left(\frac{378}{85}, \frac{441}{85}\right)$. The point of intersection at the top is found similarly.

$$x = 0 \quad \text{and} \quad y = 9 - \frac{6}{7}x$$

$$y = 9 - \frac{6}{7}(0) = 9$$

The top point of intersection is $(0, 9)$.